THREE-STEP HYBRID BLOCK ALGORITHM FOR THE SOLUTION OF INITIAL VALUE PROBLEMS OF FIRST ORDER ORDINARY DIFFERENTIAL EQUATIONS

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Abstract:
In this paper, a three-step collocation and interpolation technique is used to solve first order initial value problems of ordinary differential equations. The three-step method was developed using Power series polynomials as the basis function, and the method was augmented by the addition of off-step points to bring zero stability and upgrade the order of consistency of the new method. The derived continuous scheme has the advantage of producing several outputs of solution at off-grid points without the need for additional interpolation. To confirm the method's efficiency and accuracy, numerical examples are solved using the MAPLE 18 software package.
Keywords: collocation; power series polynomial; interpolation; block method, off-grid points.

Introduction

An Ordinary Differential Equation (ODE) is an equation involving the function's ordinary derivatives. Differential equations are a major area of mathematics with a variety of methods and solutions. A well-posed differential equation problem consists of at least one differential equation and at least one additional equation such that the system as a whole has one and only one solution (existence and uniqueness) known as the analytic or exact solution. Furthermore, this analytic solution must be continuously dependent on the data in the sense that if the equations are slightly changed, the solution does not change significantly. The goal of this research is to find the solution to a first-order differential equation using a numerical linear multistep method Abubakar *et al.* (2014). ODEs are numerically solved using linear multistep methods. A numerical method starts from an initial point and then moves forward in time to find the next solution point. The procedure is repeated in order to map out the solution. To determine the current value, the single-step method uses only one previous point and its derivative. A method like Runge-Kutta efficiently takes some intermediate steps by keeping and using information from previous steps rather than discarding all previous information before proceeding to the next step, Adee *et al.* (2005). Some authors in the literature that solved ordinary differential equations using numerical approach includes: Awoyemi (2001),(2005), Awoyemi *et al.* (2005), Ayinde *et al.* (2015), Chu *et al.* (1987), Fatunla (1988), (1994), Jator (2007), Jator *et al.* (2009), Kayode (2005), Lambert (1973), (1991), Omar *et al* (2003), (2005), Onumanyi *et al.* (1999), (2001), to mention but few. Umar *et al.* (2019) considers interpolation and collocation of rational approximate solution to give a continuous one step non-linear method for the solution of stiff initial value problems. The continuous method was evaluated at selected grid points to give discrete methods which are implemented in predictor-corrector method. The developed methods are found to be convergent and L-stable. Ajileye *et al.* (2017) presented an implicit one-step method with three off-grid points for numerical solution of second order initial value problems of ordinary differential equation has been developed by collocation and interpolation technique. The one-step method was developed using Laguerre polynomial as basis function and the method was augmented by the introduction of off-step points in order to bring zero stability and upgrade the order of consistency of the method.

In this work, We consider first order ordinary differential equation with initial value problem of the form

$$y' = f(x, y), y(x_0) = y_0 \qquad (1)$$

on the interval [a, b] has given rise to two major discrete variable methods namely, single (one) step method and multistep methods commonly known as linear multistep method (LMMs).

The linear multistep method of a k-step for determining the sequence $\{y_n\}$ take the form of linear relationship between $y_{n+j}, f_{n+j}, j = 0,1,2,3, \dots, k$. The general linear k-step can be written as

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h^{\mu} \sum_{j=0}^{k} \beta_j f_{n+j} \qquad (2)$$

Where α_j and β_j are constants. We assume that $\alpha_k \neq 0$ and that not both α_0 and β_0 are zero.

When μ= 1, then, the linear k-step as linear multistep method can be written as;

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h \sum_{j=0}^{k} \beta_j f_{n+j}. \qquad (3)$$

we generate continuous linear multistep method with k =3 using the power series as a basis function.

Methodology

In this section, we derive a continuous representation of three-step method with three off-grid points to generate the main method and other methods required to set up the block method. We consider Power Series Polynomial of the form:

$$y(x) = \sum_{j=o}^{k} a_j x^j \qquad (4)$$

On the partition

$$a = X_0 < X_1 < \dots < X_n < X_{n+1} < \dots < X_N = b$$

On the integration interval [a, b], with a constant step size h, given by

$$h = X_{n+1} - X_n; n = 0,1, \dots N - 1.$$

Equation (4) is differentiated once to obtain equation of the form:

$Y'(x) = \sum_{j=0}^{k} a_j j x^{j-1} = f(x,y)$ (5)

We will interpolate at $x = x_{n+i}, i = 0$ in equation (4) and collocate at $x_{n+i}, i = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3$ in equation (5) so as to obtain a system of eight equations each of degree seven (i.e. k=7) of the form:

$$AX = U \tag{6}$$

where

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \end{bmatrix}^T$$

$$U = \begin{bmatrix} y_{n+q} & f_n & f_{n+\frac{1}{2}} & f_{n+1} & f_{n+\frac{3}{2}} & f_{n+2} & f_{n+\frac{5}{2}} & f_{n+3} \end{bmatrix}^T$$

$$X = \begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n 5 & x_n^6 & x_n^7 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 \\ 0 & 1 & 2x_n^2 + h & 3(x_n + \frac{1}{2}h)^2 & 4(x_n + \frac{1}{2}h)^3 & 5(x_n + \frac{1}{2}h)^4 & 6(x_n + \frac{1}{2}h)^5 & 7(x_n + \frac{1}{2}h)^6 \\ 0 & 1 & 2x_n^2 + 2h & 3(x_n + h)^2 & 4(x_n + h)^3 & 5(x_n + h)^4 & 6(x_n + h)^5 & 7(x_n + h)^6 \\ 0 & 1 & 2x_n^2 + 3h & 3(x_n + \frac{3}{2}h)^2 & 4(x_n + \frac{3}{2}h)^3 & 5(x_n + \frac{3}{2}h)^4 & 6(x_n + \frac{3}{2}h)^5 & 7(x_n + \frac{3}{2}h)^6 \\ 0 & 1 & 2x_n^2 + 4h & 3(x_n + 2h)^2 & 4(x_n + 2h)^3 & 5(x_n + 2h)^4 & 6(x_n + 2h)^5 & 7(x_n + 2h)^6 \\ 0 & 1 & 2x_n^2 + 5h & 3(x_n + \frac{5}{2}h)^2 & 4(x_n + \frac{5}{2}h)^3 & 5(x_n + \frac{5}{2}h)^4 & 6(x_n + \frac{5}{2}h)^5 & 7(x_n + \frac{5}{2}h)^6 \\ 0 & 1 & 2x_n^2 + 6h & 3(x_n + 3h)^2 & 4(x_n + 3h)^3 & 5(x_n + 3h)^4 & 6(x_n + 3h)^5 & 7(x_n + 3h)^6 \end{bmatrix}$$

We solve equation (6) using MAPLE 18 software package to obtain the value of the unknown parameters

$$\alpha_0 = 1$$

$$\beta_0 = th - \frac{49}{20}t^2h + \frac{406}{135}t^3h - \frac{49}{24}t^4h + \frac{7}{9}t^5h - \frac{7}{45}t^6h + \frac{4}{315}t^7h$$

$$\beta_{\frac{1}{2}} = 6t^2h - \frac{58}{5}t^3h + \frac{29}{13}t^4h - \frac{62}{15}t^5h + \frac{8}{9}t^6h - \frac{8}{105}t^7h$$

$$\beta_1 = -\frac{15}{2}t^2h + \frac{39}{2}t^3h - \frac{461}{24}t^4h + \frac{137}{15}t^5h - \frac{19}{9}t^6h + \frac{4}{21}t^7h$$

$$\beta_{\frac{3}{2}} = \frac{20}{3}t^2h - \frac{508}{27}t^3h + \frac{62}{3}t^4h - \frac{484}{45}t^5h - \frac{8}{3}t^6h - \frac{16}{63}t^7h$$

$$\beta_2 = -\frac{15}{4}t^2h + 11t^3h - \frac{307}{24}t^4h + \frac{107}{15}t^5h - \frac{17}{9}t^6h + \frac{4}{21}t^7h$$

$$\beta_{\frac{5}{2}} = \frac{6}{5}t^2h - \frac{18}{5}t^3h + \frac{13}{3}t^4h - \frac{38}{15}t^5h + \frac{32}{45}t^6h - \frac{8}{105}t^7h$$

$$\beta_3 = -\frac{1}{6}t^2h + \frac{137}{270}t^3h - \frac{5}{8}t^4h + \frac{17}{45}t^5h - \frac{1}{9}t^6h + \frac{4}{315}t^7h$$

Substituting the values for $a_j$and $\beta_j$into equation (4) to yields a continuous hybrid three-step method of the form:

$$Y(x) = \alpha_0(x)y_n + h[\sum_{j=0}^{3} \beta_j(x) f_{n+j} + \beta_{\frac{1}{2}}(x) f_{n+\frac{1}{2}} + \beta_{\frac{3}{2}}(x) f_{n+\frac{3}{2}} + \beta_{\frac{5}{2}}(x) f_{n+\frac{5}{2}}] \tag{7}$$

Where $\alpha_j(x) and\ \beta_j(x)$are continuous coefficient $y_{n+j} = y(x_n + jh)$ is the numerical approximation of the analytical solution at $x_{n+j} and\ f_{n+j} = f(x_{n+j}, y_{n+j}, y'_{n+j})$.

We evaluate equation (7) at $x_{n+i}, i = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3$ to give discrete schemes of six equations.

$$y_{n+\frac{1}{2}} = y_n + \frac{19087}{120960}hf_n + \frac{2713}{5040}hf_{n+\frac{1}{2}} - \frac{15487}{40320}hf_{n+1} + \frac{293}{945}hf_{n+\frac{3}{2}} - \frac{6737}{40320}hf_{n+2} + \frac{263}{5040}hf_{n+\frac{5}{2}} - \frac{863}{120960}hf_{n+3} \tag{8}$$

$$y_{n+1} = y_n + \frac{1139}{7560}hf_n + \frac{47}{63}hf_{n+\frac{1}{2}} + \frac{11}{2520}hf_{n+1} + \frac{166}{945}hf_{n+\frac{3}{2}} - \frac{269}{2520}hf_{n+2} + \frac{11}{315}hf_{n+\frac{5}{2}} - \frac{37}{7560}hf_{n+3} \tag{9}$$

$$y_{n+\frac{3}{2}} = y_n + \frac{137}{896}hf_n + \frac{81}{112}hf_{n+\frac{1}{2}} + \frac{1161}{4480}hf_{n+1} + \frac{17}{35}hf_{n+\frac{3}{2}} - \frac{729}{4480}hf_{n+2} + \frac{27}{560}hf_{n+\frac{5}{2}} - \frac{29}{4480}hf_{n+3} \tag{10}$$

$$y_{n+2} = y_n + \frac{143}{945}hf_n + \frac{232}{315}hf_{n+\frac{1}{2}} + \frac{64}{315}hf_{n+1} + \frac{752}{945}hf_{n+\frac{3}{2}} + \frac{29}{315}hf_{n+2} + \frac{8}{315}hf_{n+\frac{5}{2}} - \frac{4}{945}hf_{n+3} \tag{11}$$

$$y_{n+\frac{5}{2}} = y_n + \frac{3715}{24192}hf_n + \frac{725}{1008}hf_{n+\frac{1}{2}} + \frac{2125}{8064}hf_{n+1} + \frac{125}{189}hf_{n+\frac{3}{2}} + \frac{3875}{48064}hf_{n+2} + \frac{235}{1008}hf_{n+\frac{5}{2}} - \frac{275}{24192}hf_{n+3} \tag{12}$$

$$y_{n+3} = y_n + \frac{41}{280}hf_n + \frac{27}{35}hf_{n+\frac{1}{2}} + \frac{27}{280}hf_{n+1} + \frac{34}{35}hf_{n+\frac{3}{2}} + \frac{27}{280}hf_{n+2} + \frac{27}{35}hf_{n+\frac{5}{2}} + \frac{41}{280}hf_{n+3} \tag{13}$$

STABILITY ANALYSIS

The block method is defined by Fatunla (1988) as;

$$Y_m = \sum_{i=0}^{k} A_i + h\sum_{i=0}^{k} B_i F_{m-i}$$

$$where\ Y_m = [y_n, y_{n+1}, y_{n+2}, \dots, y_{n+r-1}]^t$$

$$F_m = [f_n, f_{n+1}, f_{n+2}, \dots, f_{n+r-1}]^t$$

$A_i's\ and\ B_i's$ are chosen r x r matrix coefficient and $m = 0,1,2$ ... represents the block number, $n = mr$, the first step number in the m-th block and r is the proposed block size.

The block method is said to be zero stable if the roots of $R_j j = 1(1)k$ of the first characteristics polynomial is

$$\rho(R) = \det\left[\sum_{i=0}^{k} A_i R^{k-1}\right] = 0, A_0 = I$$

Satisfies $|\mathbf{R_j}| \leq \mathbf{1}$, if one of the roots is +1, then the root is called Principal Root of $\rho(R)$.

$$A^{(0)}=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}$$

Analysis of Zero Stable for schemes

$$L\{y(x);h\}=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+\frac{1}{2}}\\y_{n+1}\\y_{n+\frac{3}{2}}\\y_{n+2}\\y_{n+\frac{5}{2}}\\y_3\end{bmatrix}-\begin{bmatrix}0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+\frac{1}{2}}\\y_n\\y_{n-\frac{1}{2}}\\y_{n+1}\\y_{n-\frac{3}{2}}\\y_{n+2}\end{bmatrix}$$

$$-h\begin{bmatrix}\frac{2713}{5040}&\frac{-15487}{40320}&\frac{293}{945}&\frac{-6737}{40320}&\frac{263}{5040}&\frac{-863}{120960}\\\frac{47}{63}&\frac{11}{2520}&\frac{166}{945}&\frac{-269}{2520}&\frac{11}{315}&-\frac{37}{7560}\\\frac{81}{112}&\frac{1161}{4480}&\frac{17}{35}&-\frac{729}{4480}&\frac{27}{560}&-\frac{29}{4480}\\\frac{232}{315}&\frac{64}{315}&\frac{752}{945}&\frac{29}{325}&\frac{8}{315}&-\frac{4}{945}\\\frac{725}{1008}&\frac{2125}{8064}&\frac{125}{189}&\frac{3875}{48064}&-\frac{275}{24192}&-\frac{275}{24192}\\\frac{27}{35}&\frac{27}{280}&\frac{34}{35}&\frac{27}{280}&\frac{27}{35}&\frac{41}{280}\end{bmatrix}\begin{bmatrix}f_{n+\frac{1}{2}}\\f_{n+1}\\f_{n+\frac{3}{2}}\\f_{n+2}\\f_{n+\frac{5}{2}}\\f_{n+3}\end{bmatrix}$$

Where,

$$A^{(0)}=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix},\quad e=\begin{bmatrix}0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\\0&0&0&0&0&1\end{bmatrix}\quad d=\begin{bmatrix}0&0&0&0&0&\frac{19087}{120960}\\0&0&0&0&0&\frac{1139}{7560}\\0&0&0&0&0&\frac{137}{896}\\0&0&0&0&0&\frac{143}{945}\\0&0&0&0&0&\frac{3715}{2492}\\0&0&0&0&0&\frac{41}{280}\end{bmatrix}$$

$$b=\begin{bmatrix} \frac{2713}{5040} & \frac{-15487}{40320} & \frac{293}{945} & \frac{-6737}{40320} & \frac{263}{5040} & \frac{-863}{120960} \\ \frac{47}{63} & \frac{11}{2520} & \frac{166}{945} & \frac{-269}{2520} & \frac{11}{315} & -\frac{37}{7560} \\ \frac{81}{112} & \frac{1161}{4480} & \frac{17}{35} & -\frac{729}{4480} & \frac{27}{560} & -\frac{29}{4480} \\ \frac{232}{315} & \frac{64}{315} & \frac{752}{945} & \frac{29}{325} & \frac{8}{315} & -\frac{4}{945} \\ \frac{725}{1008} & \frac{2125}{8064} & \frac{125}{189} & \frac{3875}{48064} & -\frac{275}{24192} & -\frac{275}{24192} \\ \frac{27}{35} & \frac{27}{280} & \frac{34}{35} & \frac{27}{280} & \frac{27}{35} & \frac{41}{280} \end{bmatrix}$$

The first characteristics polynomial of the scheme is

$\rho(\lambda) = \det[\lambda A^0 - e]$

$$\rho(\lambda) = det[\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}]$$

$$\rho(\lambda) = det \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 & -1 \\ 0 & \lambda & 0 & 0 & 0 & -1 \\ 0 & 0 & \lambda & 0 & 0 & -1 \\ 0 & 0 & 0 & \lambda & 0 & -1 \\ 0 & 0 & 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & \lambda-1 \end{bmatrix} \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 & -1 \\ 0 & \lambda & 0 & 0 & 0 & -1 \\ 0 & 0 & \lambda & 0 & 0 & -1 \\ 0 & 0 & 0 & \lambda & 0 & -1 \\ 0 & 0 & 0 & 0 & \lambda & -1 \\ 0 & 0 & 0 & 0 & 0 & \lambda-1 \end{bmatrix} = 0$$

$\lambda^6(\lambda - 1) = 0$

$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 0 \; or \; \lambda_6 = 1$

We can see clearly that no root has modulus greater than one (i.e. $\lambda \leq 1$). The hybrid block method is zero stable.

CONVERGENCE

Zero stability and consistency are sufficient conditions for a linear multistep method to be convergent. Hence, since our hybrid block method is zero stable and consistent, it can be concluded that our method is convergent for all our cases.

REGION OF ABSOLUTE STABILITY (RAS)

A numerical integrator is said to be A-stable if its region of absolute stability R incorporate the entire left hand of the complex plane denoted by C i.e. $R = \{z \in C / re(z) < 0\}$

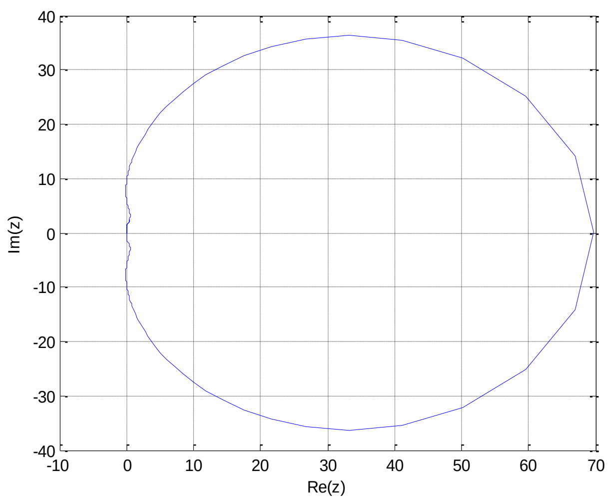
We shall adopt the boundary locus method to determine the region of absolute stability of our method. The stability polynomial is gotten using scientific workplace software. This gives

$$\bar{h}(w) = -h^6\left(\left(\frac{268309}{(88670400)}\right)w^6 - \left(\frac{336771817}{(129931692800)}\right)w^4\right) + h^5\left(\left(\frac{(9818891)}{(295568000)}\right)w^5 + \left(\frac{(19113942517)}{(38979507840000)}\right)w^5\right)$$

$$-h^4\left(\left(\frac{(63593489)}{(29556800)}\right)w^6\right) - h^3\left(\left(\frac{(735217494737)}{(2386500480000)}\right)w^6 + \left(\frac{(2048400547)}{(13300560000)}\right)w^6\right)$$

$$+h^2\left(\left(\frac{(4895537)}{(11083800)}\right)w^6 - \left(\frac{(166984692397)}{(139212528000)}\right)w^5\right) - h\left(\left(\left(\frac{2533}{1920}\right)\right)\right)w^6 + \left(\left(\frac{((3227))}{(1920)}\right)\right)w^5 + w^6 - w^5$$

The Region of Absolute Stability of our method is plotted using MATLAB 2010 version.



NUMERICAL ILLUSTRATION

In order to confirm the efficiency and accuracy of the proposed method, we considered two problems. Our results from the proposed methods are compared with the existing methods. All calculations and programs are carried out with the aid of Maple Software.

Problem 1

$y' = y, y(0) = 1, h = 0.1$

Exact Solution: $y(x) = \exp(x)$

Source:Ayinde *et al.* (2015)

Problem 2

$y' = -y, y(0) = 1, h = 0.1$

Exact Solution: $y(x) = \exp(-x)$

Source:Abubakar *et al.* (2014)

Table 1: The exact solution and the computed results from the proposed method for problem 1

x	Exact Solution	New Method	Error in New method	Error in Ayinde *et al* (2015)
0.1	1.105170918075650	1.105170918075220	4.30000e-13	1.22622104e-005
0.2	1.221402758160170	1.221402758159720	4.50000e-13	1.35518383e-005
0.3	1.349858807576000	1.349858807575870	1.30000e-13	1.49770976e-005
0.4	1.491824697641270	1.491824697640550	7.20000e-13	1.655225270e005
0.5	1.648721270700130	1.648721270699350	7.80000e-13	1.82930683e-005
0.6	1.822118800390510	1.822118800390160	3.50000e-13	2.02169671e-005
0.7	2.013752707470480	2.013752707469320	1.100000e-13	2.23432041e-005
0.8	2.225540928492470	2.225540928491200	1.270000e-13	2.46930594e-005
0.9	2.459603111156950	2.459603111156240	7.10000e-13	2.72900511e-005
1.0	2.718281828459050	2.718281828457210	1.840000e-13	3.016017084e-005

Table 2: The exact solution and the computed results from the proposed method for problem 2

x	Exact Solution	New Method	Error in New method	Error in Abubakar *et al* (2014)
0.1	0.904837418035960	0.904837418035712	2.480000e-13	3.60E-11
0.2	0.818730753077982	0.818730753077754	2.280000e-13	4.22E-06
0.3	0.740818220681718	0.740818220681788	7.00000e13	7.6E-06
0.4	0.670320046035639	0.670320046035519	1.20000e-13	1.03E-05
0.5	0.606530659712633	0.606530659712522	1.11000e-13	1.24E-05
0.6	0.548811636094026	0.548811636094132	1.06000e-13	1.41E-05
0.7	0.496585303791410	0.496585303791369	4.1000e-14	1.52E-05
0.8	0.449328964117222	0.449328964117183	3.9000e-14	1.15E-05
0.9	0.406569659740599	0.406569659740716	1.17000e-13	1.66E-O5
1.0	0.367879441171442	0.367879441171447	5.0000e-15	1.69E-05

Conclusion

In this paper, we developed a block method with three hybrid points for the solution of first-order initial value problems in ordinary differential equations. Our method was found to be efficient and accurate. The numerical results as shown in tables 1 and 2 indicate that our method is computationally reliable and gives better accuracy than the existing methods.

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